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That the above formula does not give the volume bounded by  $f(x, y, z) = 0$ , the  $xy$ -plane and a cylinder whose elements are parallel to the  $z$ -axis, may be readily seen by applying it to the plane  $z = c$ , in which case it gives a result one-third as large as the correct result.

Also solved by the Proposer.

239 (Number Theory) [March, 1916]. Proposed by HAROLD T. DAVIS, Colorado Springs, Colorado.

Give a general method for determining the solution in integers of the equation

$$x^r - 10xy - (n + 1) + y = 0,$$

where  $n$  and  $r$  are positive integers.

SOLUTION BY ELIJAH SWIFT, University of Vermont.

Solving for  $y$ ,  $y = \frac{x^r - (n + 1)}{10x - 1}$ , which must be an integer. Since the denominator is prime to 10 and the numerator integral,  $y$  will be an integer if  $\frac{10^r x^r - (n + 1) \cdot 10^r}{10x - 1}$  is. Dividing algebraically the remainder is  $1 - (n + 1) \cdot 10^r$ . If then  $\frac{10^r(n + 1) - 1}{10x - 1}$  is an integer, so is  $y$ . Hence the general process (perhaps not that desired) is the following: form  $10^r(n + 1) - 1$  and factor it. Equate  $10x - 1$  to any factor whose last digit is 9, and we have an integral solution.

261 (Number Theory) [March, 1917]. Proposed by NORMAN ANNING, Chilliwack, B. C.

Show that for any positive integer  $n$  (excluding powers of 2) positive integers  $a_1, a_2, a_3, \dots, a_k$  which are less than  $n/2$  can be chosen in such a way that

$$2^k \cos(a_1\pi/n) \cos(a_2\pi/n) \cos(a_3\pi/n) \cdots \cos(a_k\pi/n) = 1.$$

SOLUTION BY C. F. GUMMER, Queen's University.

Since  $n$  is not a power of 2, it is of the form  $(2k + 1)l$ . The equation

$$\cos(2k + 1)x - \cos(2k + 1)\alpha = 0$$

has  $2k + 1$  distinct roots in  $\cos x$ , when  $\cos(2k + 1)\alpha \neq 1$ , the roots being

$$\cos\left(\alpha + \frac{2i\pi}{2k + 1}\right), \quad i = 0, 1, \dots, 2k.$$

Also  $\cos(2k + 1)x = 2^{2k} \cos^{2k+1} x - \dots$ , the absolute term being zero. Hence,

$$2^{2k} \prod_{i=0}^{2k} \cos\left(\alpha + \frac{2i\pi}{2k + 1}\right) = \cos(2k + 1)\alpha.$$

By taking the limit of each side when  $\alpha \rightarrow 0$ ,

$$2^{2k} \prod_{i=0}^{2k} \cos \frac{2i\pi}{2k + 1} = 1,$$

that is,

$$\left\{ 2^k \prod_{i=0}^k \cos \frac{2i\pi}{2k + 1} \right\}^2 = 1,$$

or

$$2^k \prod_{i=0}^k \cos \frac{2i\pi}{2k + 1} = \pm 1.$$

By taking  $j = 2i$ , when  $i \leq k/2$  and  $j = 2k + 1 - 2i$  when  $i > k/2$ , we get

$$2^k \prod_{j=1}^k \cos \frac{j\pi}{2k + 1} = \pm 1,$$

which takes the required form if  $a_j = jl$ .